Application of Compound Compressible Flow to Nonuniformities in Hypersonic Propulsion Systems

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An analytical model is presented to study the propagation of the forebody boundary layer inside the engine channel of a hypersonic engine, based on a compound compressible one-dimensional streamtube method. It is shown that the compound compressible flow model can be greatly simplified in the hypersonic regime. This model is appropriate for studying the effects of nonuniform flow inside a supersonic combustion ramjet. It is found that even a small boundary layer can produce noticeable changes in freestream properties inside the inlet. In the combustor models, it is seen that freestream Mach number is generally increased, and so static pressure and temperature are decreased. It is suggested that these effects will reduce the combustion reaction rate relative to that expected in a uniform flow and will therefore reduce the total heat release and increase losses inside the engine.

Nomenclature

A = streamtube area
$\mathcal{A}$ = streamtube area ratio
f = Cohen and Reshotko stream function
H = channel height
M = Mach number
$\mathfrak{M}$ = relative logarithmic derivative of Mach number
n = profile power law exponent
P = static pressure
R = gas constant
S = Cohen and Reshotko enthalpy function
T = static temperature
T_o = total temperature
U = velocity
u = streamwise velocity component
x = coordinate along flow direction
y = coordinate normal to wall and flow direction
z = coordinate along wall and normal to flow direction
β = compound choking parameter
Γ = total temperature parameter
γ = ratio of specific heats
δ = boundary-layer thickness
δ^* = boundary-layer displacement thickness
η = transformed normal coordinate
λ = characteristic channel length scale
ρ = density
τ = shear
$\overline{\tau_w'}$ = wall shear function
Υ = compound Mach number parameter

Subscripts

D = design value
e = freestream
i = current streamtube
j = other streamtube
δ = boundary-layer value

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I. Introduction

Boundary Layers on Hypersonic Vehicles

THE extreme Mach numbers at which supersonic combustion ramjets, or scramjets, are to operate exceed our current experience with air-breathing engines. Current wind-tunnel technology permits testing to about Mach 8, although actual air-breathing flight experience extends only somewhat above Mach 3. It is difficult to extrapolate these supersonic results to the hypersonic regime because of the increasing importance of real gas effects in the strong gradients and high temperatures encountered at very high Mach number. Thus, until hypersonic test vehicles can be flown, there will be a substantial gap between the knowledge required to successfully build and operate such a vehicle and existing hardware experience.

In typical configurations, a scramjet would be set far back on a hypersonic vehicle, behind a long forebody, as determined by the geometric requirements for matching the close-fitting bow shock to the scramjet inlet, while permitting adequate mass ingestion. This means that flow entering the scramjet inlet will have passed over a substantial length of the vehicle, leading to the possible formation, under certain circumstances, of a thick boundary layer. Indeed, hypersonic boundary layers are thick in general, because the high temperatures associated with decelerated high-speed flow create a very low-density region and lead to increased viscosity.[1] Because of their low density, hypersonic boundary layers carry very little mass flux in relation to the inviscid freestream.

There is much uncertainty over whether the forebody will be laminar or turbulent because data on hypersonic transition are scarce, and the ground test data that do exist conflict with flight test data. Published steady-state wind-tunnel transition data extend to Mach 8,[2] with transient test data to Mach 16.[3,4] Some estimates of transition length from the cone data of Sheetz are available in Ref. 5. On a trajectory designed to provide one atmosphere dynamic pressure, it is estimated that a 30-m forebody with small wedge angle will be fully laminar above Mach 17. Because of the fullness of the turbulent profile, the laminar boundary layer has a more dramatic effect on properties in the freestream and so represents a worst-case scenario in studying nonuniformity effects on the freestream combustion process.

Classical Laminar Boundary-Layer Treatment

The results of the classical boundary-layer theory of Bertram and Blackstock[6] have been applied to estimate laminar boundary-layer displacement thickness on a hypersonic forebody, as described in Ref. 7. Important observations from these results, which demonstrate the conclusions of classical hypersonic theory, are as follows:

1) The boundary-layer thickness increases dramatically with altitude (because kinematic viscosity ν increases nearly exponentially in the atmosphere).

2) Large positive angles of attack drastically reduce the boundary-layer thickness, an effect that is linear with angle of attack at large angle of attack.

3) The bow shock is displaced from where it would be in an inviscid flow.

4) Wall cooling and real-gas effects reduce the boundary-layer size.

Figure 1 presents contours indicating the percentage of the inlet flow that is represented by the boundary layer 10 m along the forebody. The boundary layer is referenced to the shock-layer height H, which is the height of the shock above the surface and equals the sum of the boundary-layer height, δ, and the inviscid shock layer. Note that for laminar boundary layers, the 99% height is approximately equal to the displacement thickness δ^*, so that these concepts can be merged.[1] These contours are calculated for vanishingly small forebody wedge angle, and assuming a straight forebody without compression ramps. The boundary-layer thickness is strongly dependent on vehicle geometry and attitude, so that these contours should be taken as a rough indication of inlet conditions only. Note that because the boundary-layer scales approximately with $\sqrt{x}$ and the shock-layer height scales approximately with x at large x, the fractional thickness at distances other than 10 m can be calculated by transforming the contours of Fig. 1 into

$$\frac{\delta}{H}(x) \simeq \frac{\sqrt{x/10}}{(1/[\delta/H|_{10}] - 1)x/10 + \sqrt{x/10})} \tag{1}$$

where x is measured in meters. Thus, at 20 m, what was the 20% contour at 10 m becomes 15%, and the 50% contour becomes 41%. At 30 m, the 20% contour at 10 m represents the 12.5% contour, and the 50% contour at 10 m becomes 37% at 30 m.

Sensitivity of Combustion to Thermodynamic Conditions

The presence of flow nonuniformities in the scramjet will be particularly important if they vary freestream thermodynamic conditions, because even small changes in Mach number can produce large changes in thermodynamic properties. Also, a scramjet combustor that is designed to run near thermal choking at uniform inlet conditions may be driven to choke with a nonuniform inlet. These effects may not only be deleterious to engine performance but may generate temperatures and pressures that destroy the engine.

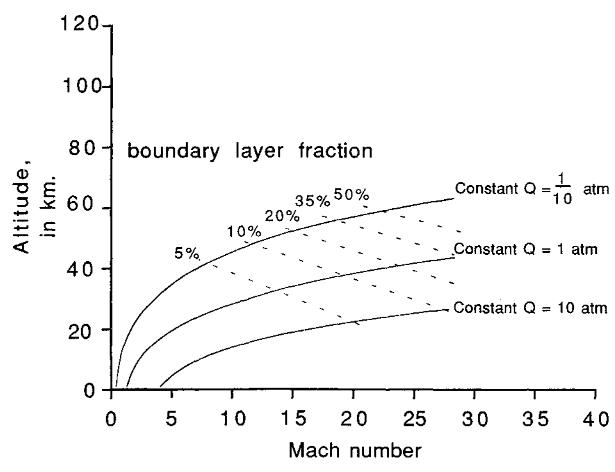

Fig. 1 Percentage of inlet flow occupied by boundary layer at 10-m wall cooled to 2000 K with small wedge angles. At large angles, thickness is reduced in inverse proportion to wedge angle.

One of the major issues in scramjet design is accomplishing combustion at high speeds in a combustor channel of reasonable length, assuming that fuel can be adequately mixed into the flow. This is particularly true for a scramjet operating at low pressure (say, <0.1 atm.), where the reaction rate may dominate the combustion process. Any process that delays or inhibits combustion in that case will either reduce the amount of heat that is added in the engine or required the design of a longer, heavier engine.

Rogers and Schexnayder[8] have presented a curve fit for the equilibrium reaction time for 95% completion between 1000 and 2000 K and 0.2 and 5 atm, which modifies the similar results of Ferri[9]:

$$t_{\text{reaction}} = \frac{325}{p^{1.6}} e^{-T/1250} \tag{2}$$

with time in μs, T in degrees K, and p in atm. The reaction time is a sensitive function of the initial static pressure and, to a lesser extent, static temperature. At high p§ressures, the reaction rate will be slowed by secondary reactions and, at high temperatures, reaction will slow as the equilibrium temperature is approached. It is therefore important that the pressure and temperature be kept within a rather narrow range once the combustor length, and thus the required reaction rate, is selected.

II. Multistreamtube One-Dimensional Model

The influence of the forebody profile on the channel properties will be examined with a streamtube model based on that of Bernstein et al.[10] The model assumes that flow is essentially one-dimensional and that adjacent streamtubes have matched static pressures. In fact, the problem can be modified to account for an imposed pressure gradient, to model the presence of shocks or curvature, with relative ease.

The justification for using a streamtube model is based on the well-known result that supersonic shear layers tend not to mix out.[11] Once a profile is formed and enters the channel of a hypersonic engine, it is reasonable to expect that it will persist well into the engine without mixing. Thus, streamtubes will retain their integrity. It is also expected that transverse pressure gradients will be small entering a hypersonic engine because both the boundary-layer transverse pressure gradient and shock-layer gradient are small. Streamtube pressure-matching is a good assumption because hypersonic engine channels will typically be of small aspect ratio, so that any transverse pressure gradients that do form, through the action of shocks or curvature, will re-equilibrate quickly down the channel.[5] This method is clearly inadequate for incorporating mixing or strong shocks in the channel solution, however, and so is most useful for indicating qualitative trends as a prelude for more detailed numerical solutions.

The advantage of solving for a pressure-matched flow will be particularly apparent with multiple streamtubes because, once the matching condition is satisfied, all streamtubes can be treated independently from the rest of the channel flow. Such a model will be valid in the channel of a supersonic combustion ramjet away from the vicinity of strong shocks, curved shocks, or regions where surface curvature produces significant centrifugal effects. The freestream and boundary layer are then modeled as one-dimensional streamtubes as they enter the engine channel. The relative areas of these streamtubes, and the Mach number and temperature profiles, can be input from the forebody boundary-layer analysis.

It is assumed that viscous effects are unimportant in evaluations of the behavior of the freestream inside the engine channel. Viscosity is required for generating the boundary-layer profile on the forebody but, once that profile is formed, further viscous effects will be neglected, except at the wall. The validity of this assumption depends on the forebody's being much longer than the channel length of the scramjet, so that the residence time of flow inside the engine is much smaller than on the vehicle surface. Also, in this model, formation and further growth of boundary layers inside the engine itself will be neglected.

Inviscid compound flow behavior has been characterized by the compound flow parameter, β,[10] which is defined in terms of the local static pressure P and channel height H as

$$\beta = \frac{dH}{dx} \bigg/ \frac{d\ell nP}{dx} \tag{3}$$

where x is the streamwise coordinate.

When β is positive, the channel is "compound subsonic," even though one or more of its streamtubes may be supersonic, meaning that the entire flow behaves as if it were subsonic, decelerating in an expanding channel and accelerating in a converging one. Conversely, when β is negatively valued, the entire flow is "compound supersonic," behaving as if it were supersonic, so that it accelerates in a diverging channel and decelerates in a converging one. At values of $\beta = 0$, the flow is "compound-choked" and will not accept further reductions in channel area. This is the limit at which information will no longer propagate upstream, even though a part of the channel flow may be subsonic, because information propagation is limited by the pressure-matching condition across the channel cross section.

In a channel with i streamtubes, because the compound flow parameter depends on each individual streamtube's area and Mach number as

$$\beta = \sum_i \frac{A_i}{\gamma_i}\left(\frac{1}{M_i^2} - 1\right) \tag{4}$$

a streamtube with very low Mach number, and thus a very large value of $1/M^2$, can dominate the behavior of the entire channel flow, even though its area is small.

Conversely, great simplification is possible at hypersonic conditions:

$$\left(\frac{1}{M^2} - 1\right) \simeq -1 \tag{5}$$

so that, at high Mach numbers, the compound flow parameter becomes independent of Mach number. This immediately suggests that a low Mach number streamtube, such as might be associated with a boundary layer, can dominate the channel β value, and so have a driving influence on the entire channel, if it has sufficient area. The conditions under which this will occur will be explored later.

It is convenient to define another compound-choking parameter, a uniform equivalent Mach number, which will be termed $\overline{M}$ and will be selected such that

$$\left(\frac{1}{\overline{M}^2} - 1\right)\sum_i \frac{A_i}{\gamma_i} \equiv \sum_i \frac{A_i}{\gamma_i}\left(\frac{1}{M_i^2} - 1\right) \tag{6}$$

or

$$\overline{M} = \frac{1}{\sqrt{1 + \gamma_{\text{average}}\,\beta/A_{\text{total}}}} \tag{7}$$

In other words, a streamtube with equivalent uniform Mach number $\overline{M}$ will have the same response to channel area change as the nonuniform combination of streamtubes over the same area. A nonuniform channel flow could be replaced with a uniform flow at $\overline{M}$ and have the same β *at one station*.

Although the equivalent uniform streamtube concept is the most straightforward means of comparing the nonuniform flow to a uniform streamtube, especially in examining the streamwise variation of freestream thermodynamic properties, the analogy cannot be carried beyond a single station in a channel. Changes in the value of $\overline{M}$ for a given change in channel area will not, in general, be equal to the change that would be observed in an actual uniform streamtube. Therefore, it would not be valid to calculate a channel flow by determining $\overline{M}$ at the inlet station, then replacing the entire flow by a uniform streamtube at that Mach number and following it as a one-dimensional streamtube.

Two-Streamtube Analytical Model

Because a two-stream model is the simplest compound flow, it is appropriate to examine the behavior of such a model in the hypersonic limit, although the multistreamtube model can be applied to any number of streamtubes. The two-streamtube flow will contain all of the compound compressible flow behavior evident in more sophisticated models. Throughout the rest of this work, multiple-streamtube behavior will be compared to analogous two-streamtube flows. The model will subsequently be extended to a continuous profile, in the limit of an infinite number of streamtubes, with corrections to allow for a no-slip condition at a wall.

With two inviscid streamtubes, in a channel with area change but no heat addition, the individual area change of a streamtube is found by equating the logarithmic derivatives of static pressure across the channel. In terms of the total area change, the Mach number in both streams, and the ratio of the stream areas at each point in the channel, the area change of stream 2 (which will henceforth refer to a hypersonic flow) is

$$\frac{dA_2}{A_2} = \left\{\frac{1}{1 + (M_2^2/M_1^2)\,[(1 - M_1^2)/(1 - M_2^2)\mathcal{A}]}\right\}\frac{dA}{A_2} \tag{8}$$

where $\mathcal{A} \equiv A_1/A_2$ and $A = A_1 + A_2$, the total channel area.

If M_2 is hypersonic ($\lim_{M\to\infty}$), the area derivative becomes

$$dA_2 = \left[\frac{M_1^2}{M_1^2 - (1 - M_1^2)\mathcal{A}}\right] dA \tag{9}$$

independent of M_2.

For small values of M_1, the hypersonic stream does not experience much area change. Indeed, for values of $M_1^2 < \mathcal{A}/(\mathcal{A} - 1)$ the hypersonic stream experiences an area change of the opposite sign to that of the channel. This is a result that was identified by Bernstein et al.[10] from wave analysis. With uniform γ, the influence of each stream is proportional to $A\,[(1/M^2) - 1]/\gamma$, so that a low-velocity stream can drive the behavior of the entire flow if it occupies a sufficiently large portion of the channel. The low-velocity stream experiences area change:

$$dA_1 = \left[\frac{-(1 - M_1^2)\mathcal{A}}{M_1^2 - (1 - M_1^2)\mathcal{A}}\right] dA \tag{10}$$

If $\mathcal{A} > M_1^2/(1 - M_1^2)$, the low-velocity stream is most influential in characterizing the behavior of all the fluid in the channel. This dependence is an expression of the compound flow behavior outlined by Bernstein et al., simplified in the hypersonic limit. If the low-velocity stream is actually subsonic (as part of the boundary-layer flow will be), the channel behavior can be summarized as follows:

1) If $\mathcal{A} > M_1^2/(1 - M_1^2)$, the entire flow behaves as if it were subsonic, even though part of the flow is supersonic.

2) If $\mathcal{A} < M_1^2/(1 - M_1^2)$, the entire flow behaves as if it were supersonic.

3) If $\mathcal{A} = M_1^2/(1 - M_1^2)$, the entire flow is choked, even though neither of the individual streamtubes is sonic.

On the other hand, if the low-velocity streamtube is supersonic, the compound flow will always behave as if it were supersonic, although with an effective Mach number that is lower than the hypersonic freestream value.

Because static pressures are coupled, both streamtubes must accelerate or decelerate together. This can be demonstrated rigorously. First, a parameter is defined here to compare the behavior of Mach number in one of the double streams with a single stream in the same channel:

$$\mathfrak{M} \equiv \left\{\frac{[\mathrm{d}M^2/M^2]_{\text{double}}}{[\mathrm{d}M^2/M^2]_{\text{single}}}\right\} \tag{11}$$

From the influence coefficient expression for Mach number in terms of area change, and the individual stream area change, the parameter, Eq. (11), can be written for stream i in terms of stream j as

$$\mathfrak{M}_i \equiv \left\{\frac{(1-M_i^2)A_j + (1-M_i^2)A_i}{[(M_i^2/M_j^2) - M_i^2]A_j + (1-M_i^2)A_i}\right\} \tag{12}$$

For stream 2, in the hypersonic limit,

$$\mathfrak{M}_2 \cong \left[\frac{-1}{(1/M_1^2)(A_1/A) - 1}\right] \tag{13}$$

$$\frac{\mathfrak{M}_1}{\mathfrak{M}_2} \cong \left[\frac{(M_1^2 - 1)}{M_1^2}\right] \tag{14}$$

so that $\mathfrak{M}_1/\mathfrak{M}_2 < 0$ if $\mathfrak{M}_1 < 1$, and $\mathfrak{M}_1/\mathfrak{M}_2 > 0$ if $\mathfrak{M}_1 > 1$.

In a converging channel,

$$\frac{\mathrm{d}M_1^2}{\mathrm{d}(A_1/A)} \propto \left[\frac{A_1}{A} - M_1^2\right] \tag{15}$$

so that the two-stream flow in a converging channel will move toward compound choking. This is especially important in a scramjet, where a thick boundary layer may choke the entire flow in the inlet, even though the hypersonic region has not decelerated much.

Analytical Model with Heat Addition

The effect of heat addition can be added to the two-stream model most simply by assuming total temperature changes only in the high-speed stream. This is a reasonable approximation because the high temperatures and low density of the boundary layer will preclude the addition of much heat, as has been found in numerical studies.[5] Introducing the influence of heat addition into the static presure equation for the hypersonic stream, and again equating static pressure in both streams,

$$\mathrm{d}A_2 = \left[\frac{1}{1 + M_2^2/M_1^2[(1-M_1^2)/(1-M_2^2)]\mathcal{A}}\right]\mathrm{d}A + \left[\frac{1 + [(\gamma-1)/2]M_2^2}{M_1^2/M_2^2[(1-M_2^2)/(1-M_1^2)] + \mathcal{A}}\right]\frac{A_1\,\mathrm{d}T_o}{T_o} \tag{16}$$

In the hypersonic limit,

$$\mathrm{d}A_2 = \left[\frac{M_1^2}{M_1^2 - (1-M_1^2)\mathcal{A}}\right]\mathrm{d}A - \left[\frac{1-M_1^2}{M_1^2 - (1-M_1^2)\mathcal{A}}\right]\frac{A_1\,\mathrm{d}T_o}{T_o} \tag{17}$$

Thus, even if the channel area is constant ($\mathrm{d}A \to 0$), heat addition will change the relative areas of the streams.

The influence of a boundary layer on the freestream flow behavior depends on whether that boundary layer is best represented by a subsonic or supersonic streamtube under the equivalent Mach number definition. The resolution of this issue depends on an extension of the multiple-streamtube model to a continuous profile.

III. Application to Constant-Area Combustor

The two-stream model presented in Sec. II is instructive only for identifying basic compound compressible flow trends. For realistic modeling, more streamtubes must be used, and that will be the subject of following sections. However, it is instructive to apply the results of the two-stream model to a simple combustor geometry, with the goal of identifying qualitative trends.

The effect of a low-speed region in a channel flow with heat addition can be illustrated analytically with the example of a combustor designed for constant-pressure operation with uniform inflow. In such a channel, the Mach number decreases in response to heat addition, but the area is selected such that static pressure and velocity remain constant. An advantage of this design is that there is no adverse pressure gradient to separate the boundary layer, so that losses due to heat addition are acceptable.[5]

The uniform-flow constant-pressure area and heating profiles can be found easily from the influence coefficient expression for pressure:

$$\frac{\mathrm{d}P}{P} = \left[\frac{\gamma M^2}{1-M^2}\right]\frac{\mathrm{d}A}{A} - \left\{\frac{\gamma M^2[1 + (\gamma - 1/2)]M^2}{1-M^2}\right\}\frac{\mathrm{d}T_o}{T_o} \tag{18}$$

where $\mathrm{d}T = \mathrm{d}T_o$ if velocity is constant. Constant pressure $\mathrm{d}P/P = 0$ can be satisfied by the condition

$$\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{\bar{A}}{\lambda_D}e^{x/\lambda_D} \tag{19}$$

$$\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{\bar{T}}{\lambda_D}e^{x/\lambda_D} \tag{20}$$

Overlined values represent initial conditions, and λ_D is a design length scale that characterizes the rate at which heat is added to the flow. The corresponding Mach number and total temperature profiles will be

$$\frac{\mathrm{d}M^2}{\mathrm{d}x} = \frac{\bar{M}^2}{\lambda_D}e^{-x/\lambda_D} \tag{21}$$

$$\frac{\mathrm{d}T_o}{\mathrm{d}x} = \frac{\bar{T}_o}{\lambda_D}\left(1 + \frac{\gamma-1}{2}M^2\right)^{-1}e^{x/\lambda_D} \tag{22}$$

The behavior of nonuniform flow with a low-speed streamtube is explored by inserting the profiles for area [Eq. (19)] and total temperature [Eq. (22)] into the two-stream model of Eq. (16). For convenience, define

$$\Upsilon \equiv \frac{M_1^2(1-M_2^2)}{M_2^2(1-M_1^2)} \tag{23}$$

$$\Gamma \equiv \frac{1 + (\gamma - 1/2)\bar{M}_2^2}{1 - (\gamma - 1/2)M_2(x)^2} \tag{24}$$

Υ is related to the Mach number part of the β parameter, and Γ is the proportionality of the logarithmic derivative of total temperature to that of static temperature. In the particular case of total area change being a small fraction of the initial area, the area change for the high-speed streamtube is, then,

$$\frac{\mathrm{d}A_2}{A_2} = \left\{\frac{1 + \mathcal{A}(1 + \Gamma/\Upsilon)}{1 + \mathcal{A}/\Upsilon}\right\}\frac{\mathrm{d}x}{\lambda_D} \tag{25}$$

The pressure follows:

$$\frac{\mathrm{d}P}{P} = \left[\frac{\gamma M_1^2}{1-M_1^2}\right]\left\{\frac{1 + \mathcal{A} - \Gamma}{\mathcal{A} + \Upsilon}\right\}\frac{\mathrm{d}x}{\lambda_D} \tag{26}$$

and the high-speed streamtube Mach number behaves as

$$\frac{\mathrm{d}M_2^2}{M_2^2} = \left[\frac{1}{1-M_2^2}\right]\left\{\left(1 + \gamma M_2^2 - \frac{2[1 + (\gamma - 1/2)M_2^2]\mathcal{A}}{\mathcal{A} + \Upsilon}\right)\Gamma - \frac{2[1 + (\gamma - 1/2)M_2^2]\Upsilon(\mathcal{A} + 1)}{\mathcal{A} + \Upsilon}\right\}\frac{\mathrm{d}x}{\lambda_D} \tag{27}$$

The static temperature can be related to the total temperature and area profiles as well:

$$\frac{\mathrm{d}T}{T}=\left[\frac{M_2^2}{1-M_2^2}\right]\left\{(\gamma-1)\left(\frac{1+\mathcal{A}(1+\Gamma/\Upsilon)}{1+\mathcal{A}/\Upsilon}\right)+\frac{(1-\gamma M_2^2)}{M_2^2}\Gamma\right\}\frac{\mathrm{d}x}{\lambda_D} \tag{28}$$

The Mach number, pressure, and temperature distributions can be written as

$$\frac{\mathrm{d}M_2^2}{M_2^2}=\frac{\mathrm{d}x}{\lambda_M} \tag{29}$$

$$\frac{\mathrm{d}P}{P}=\frac{\mathrm{d}x}{\lambda_P} \tag{30}$$

$$\frac{\mathrm{d}T}{T}=\frac{\mathrm{d}x}{\lambda_T} \tag{31}$$

where, for uniform inflow conditions, $\lambda_M = -\lambda_D$, $\lambda_T = \lambda_D$, and $\lambda_P = \infty$, corresponding to constant pressure, The values of λ characterize the activity length of thermodynamic changes. For instance, a small λ_M means that the Mach number is behaving as if the channel were shortened, with flow properties changing quickly. Similarly, large λ_M corresponds to a flow with very gradual changes in Mach number. Negative λ_M means that Mach number is decreasing, whereas positive values refer to increasing Mach number.

Because M_2 is a function of distance along the combustor channel, it is difficult to examine the behavior of the nonuniform flow at arbitrary x. However, at or near the combustor entrance, $e^{-x/\lambda_D} \simeq 1$ and, assuming the high-speed stream Mach number equals the original design-value uniform-flow Mach number, $M_2 \simeq \bar{M}$, so that $\Gamma \simeq 1$. The characteristic lengths, in terms of the original design length scale, are, then,

$$\frac{\lambda_D}{\lambda_M}=\left[\frac{M_2^2}{1-M_2^2}\right]\left\{\frac{1}{M_2^2}+\gamma-\left(\frac{[(2/M_2^2)+\gamma-1][\mathcal{A}(1+\Upsilon)+\Upsilon]}{\mathcal{A}+\Upsilon}\right)\right\} \tag{32}$$

$$\frac{\lambda_D}{\lambda_P}=\left[\frac{M_1^2}{1-M_1^2}\right]\left\{\frac{\gamma\mathcal{A}}{\mathcal{A}+\Upsilon}\right\} \tag{33}$$

$$\frac{\lambda_D}{\lambda_T}=\left[\frac{M_2^2}{1-M_2^2}\right]\left\{(\gamma-1)\left(\frac{1+\mathcal{A}(1+1/\Upsilon)}{1+\mathcal{A}/\Upsilon}\right)+\frac{(1-\gamma M_2^2)}{M_2^2}\right\} \tag{34}$$

In the hypersonic limit, these length scales are independent of the high-speed Mach number.

If the low-speed streamtube is subsonic, the compound compressible flow can experience either compound-subsonic, compound-choked, or compound-supersonic behavior. On the other hand, if both streamtubes are supersonic, only compound-supersonic behavior is possible. Although the properties of the flow with a subsonic streamtube will be considered in detail, it is found that the proper representation of the boundary-layer profile is with a supersonic streamtube for most cases.

Figure 2 shows the values of the characteristic pressure length and characteristic Mach number length as a function of low-speed/high-speed area ratio, with low-speed streamtube at $M_1 = 0.5$ and high-speed $M_2 = 10$. In this two-stream flow, the channel can be compound supersonic, subsonic, and choked. In accordance with Eq. (33), the characteristic pressure length scale approaches ∞ as the area ratio goes to zero. This is the uniform flow limit and experiences the consistent constant-pressure behavior. The pressure scale $\lambda_P = 0$ when the low-speed streamtube has thickness such that $\mathcal{A} = -\Upsilon$. This corresponds to the compound-choking condition found earlier. At a given low-speed Mach number, the characteristic pressure length scale is negative between the uniform-flow and compound choking limits. The two-stream flow would therefore experience a favorable streamwise pressure gradient and corresponding acceleration. For low-speed streamtubes that are thicker than the choking criterion, the pressure gradient is finite and positive, acting against the direction of flow. As stated earlier, this behavior is insensitive to Mach number at high Mach number. In fact, for M_2 greater than about 4, this plot is essentially independent of M_2.

As shown in Fig. 2, the characteristic Mach number length λ_M is also zero at the compound-choking point. In the uniform limit, $\lambda_M/\lambda_D = -1$, as expected. However, between the uniform limit and the compound-choking point, there is a region where $\lambda_M \to \infty$, corresponding to constant Mach number. From Eq. (32), this point occurs at an area ratio of

$$\mathcal{A}=\frac{M_1^2(M_2^2-1)}{M_2^2[(\gamma-2)M_1^2+1]+2M_1^2} \tag{35}$$

With $M_1 = 0.5$, $M_2 = 10$, $\gamma = 1.3$, $\lambda_M \to \infty$ at $\mathcal{A} = 0.2982$. At this point, the nonuniform flow will tend to remain at its initial Mach number rather than decreasing exponentially as did the uniform stream. Similarly, the area ratio at which $\lambda_T \to \infty$ is

$$\mathcal{A}=\frac{M_1^2(M_2^2-1)}{M_2^2[(\gamma-2)M_1^2+1]} \tag{36}$$

At the same conditions, $\lambda_T \to \infty$ at $\mathcal{A} = 0.3$. The temperature behaves qualitatively like the Mach number, with the same choking point and a slightly different constant-value area ratio.

The Mach number has three distinct types of behavior. Between the uniform limit of $\mathcal{A} = 0$ and the constant Mach number point, λ_M is negative and has greater magnitude than the design value λ_D, and magnitude increases monotonically with area ratio. In this regime, Mach number is decreasing, and the rate of that decrease is falling with increasing area ratio. Between the constant Mach number point and compound choking, λ_M is possible, corresponding to an increasing Mach number. The rate of this increase grows with increasing $\mathcal{A}$ until it becomes singular at the choking point. For low-speed streams that are thicker than the choking criterion, corresponding to compound-subsonic flow, $\lambda_M < 0$, so that Mach number is decreasing at a rate that decreases with increasing area ratio.

The area ratios at which compound choking and constant Mach number occur both depend on the magnitude of the

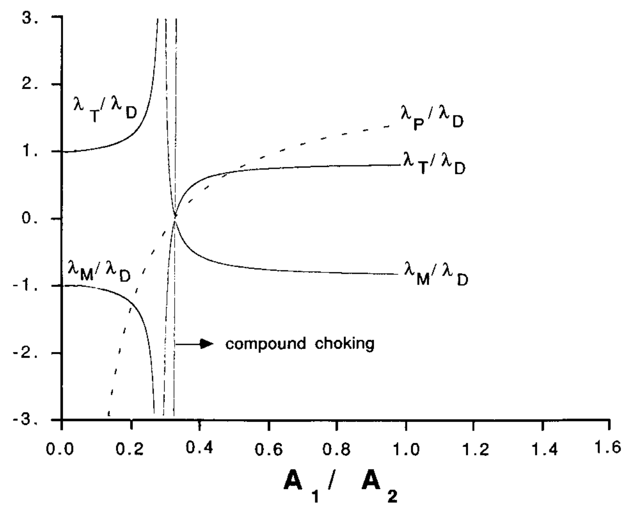

Fig. 2 Characteristic Mach number, temperature, and pressure lengths for two streamtubes in a constant-pressure combustor designed for uniform flow, with $M_1 = 0.5$, $M_2 = 10$.

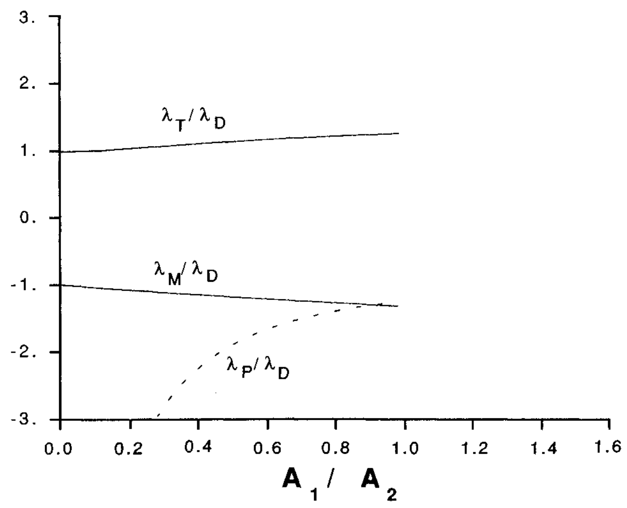

Fig. 3 Characteristic Mach number, temperature, and pressure lengths for two streamtubes in a constant-pressure combustor designed for uniform flow, with $M_1 = 2$, $M_2 = 10$.

low-speed Mach number. However, the choking condition is more sensitive, so that the spacing between these two points increases with increasing M_1. At small values of M_1 (i.e., $M_1 < 0.2$), the choking and constant Mach number points coincide, so that there is no area ratio at which Mach number in increasing down the channel.

Figure 3 is a plot of the two-streamtube behavior when both streamtubes are supersonic, with $M_1 = 2$ and $M_2 = 10$. There is no compound compressible subsonic regime because there is no subsonic streamtube, but the low-speed streamtube does still have a noticeable influence on the net channel behavior.

Finally, it is well known that heat addition to a one-dimensional flow leads to a loss in total pressure, the magnitude of which increases with increasing Mach number. However, the total pressure is not influenced by changes in area so that in the nonuniform channel, with the same relative increase in total temperature, the logarithmic derivative of total pressure is proportional only to the local Mach number. The change in losses through the combustor channel with the addition of a low-speed streamtube depends solely on the changes in Mach number. When nonuniform effects keep Mach number higher than the design conditions, the total pressure drop will be increased; conversely, if Mach number is decreased, total pressure losses are decreased as well. In fact, actual boundary-layer profiles have less dramatic effects on the freestream conditions than is indicated by the two-streamtube model because the boundary layer tends to resist changes in area.

Limitations on Discrete Streamtube Solutions

A continuous profile is represented by the compound compressible flow model in the limit of infinitely small streamtubes. The β parameter can be expressed in integral form by converting A_i, the area of each streamtube at the specific streamwise station being examined, into a differential area dA over which the integration will be carried, as per Descher[12]:

$$\beta(x) = \int_A \frac{dA}{\gamma}\left(\frac{1}{M(y,z)^2} - 1\right) \tag{37}$$

where x is the streamwise coordinate, and y and z are the planar coordinates at the given station. In two dimensions, this becomes

$$\beta = \int_0^H \left(\frac{1}{M(y,z)^2} - 1\right)\frac{dy}{\gamma} \tag{38}$$

Considering the class of solutions for profiles such that

$$M(y) = M_e\left(\frac{y}{H}\right)^n \tag{39}$$

with constant γ,

$$\beta = \frac{H^{2n} y^{(1-2n)}}{\gamma(1-2n)M_e^2} - \frac{y}{\gamma}\Bigg|_{y=0}^{y=H} \tag{40}$$

For values of $n < ½$,

$$\beta = \frac{H}{\gamma}\left(\frac{1}{(1-2n)M_e^2} - 1\right) \tag{41}$$

For small values of n, (i.e., nearly uniform profiles), the β parameter is only slightly different from the uniform flow solution and resembles a uniform flow with a slightly smaller freestream Mach number. Actual laminar boundary-layer profiles are far from uniform, with nearly linear profiles that are best represented by values of $n \simeq 1$, especially near the wall. This greatly complicates the preceding formulation, because the integral diverges ($\beta \to \infty$) for values of $n \geq ½$. This implies that, no matter how small the boundary layer, the flow is always dominated by the wall.

This result is clearly nonphysical, and it arises from the attempt of the solution is to satisfy a no-slip condition on the wall without viscosity. From the one-dimensional momentum equation,

$$\frac{du}{dx} = -\frac{1}{\rho u}\left(\frac{dp}{dx} - \frac{\partial \tau}{\partial y}\right) \tag{42}$$

accordingly, if there is no shear (i.e., inviscid flow), $du/dx \to \infty$ in the presence of a finite pressure gradient at zero velocity. This is an example of the well-known result of inviscid flow theory.

Integral Form of the β Parameter

In order to account correctly for the no-slip condition at a wall, while retaining the one-dimensional compound compressible flow model without averaging, the β parameter will be solved in integral form with the inclusion of viscous effects. The development of Bernstein et al.[10] will be modified with the addition of a shear gradient term that will exactly cancel the accelerating influence of streamwise pressure gradient when the streamwise velocity equals zero.

The one-dimensional continuity, momentum, and energy equations are combined with the equation of state to give an expression for the logarithmic derivative of pressure in terms of area change, heat addition, and friction:

$$\frac{dP}{P} = \left[\frac{-\gamma M^2}{M^2 - 1}\right]\frac{dA}{A} + \left[\frac{\gamma M^2[1 + (\gamma - 1/2)M^2]}{M^2 - 1}\right]\frac{dT_o}{T_o} - \left[\frac{1 + (\gamma - 1)M^2}{M^2 - 1}\right]\frac{\partial \tau}{\partial y}\frac{dx}{P} \tag{43}$$

This is Shapiro's influence coefficient expression for changes in pressure due to area change, heat addition, and friction, with the exception that the shear gradient has been retained in explicit form.[13]

In the absence of heat addition, $dT_o = 0$, and

$$\frac{dA}{dx} = \frac{A}{\gamma}\left[\frac{1}{M^2 - 1}\right]\left(\frac{d\ln P}{dx} - \frac{\partial \tau/\partial y}{P}\right) - A\,\frac{\partial \tau/\partial y}{P} \tag{44}$$

Since $u = 0$ at the wall, the streamwise pressure gradient is exactly canceled by the normal shear gradient. For convenience, define

$$\overline{\tau_w'} \equiv \frac{\partial \tau/\partial y}{dP/dx} \tag{45}$$

Equation (44) can be written in simpler form:

$$\frac{dA}{dx} = \frac{A}{\gamma}\left[\frac{1}{M^2} - 1\right](1 - \overline{\tau_w'})\frac{d\ell nP}{dx} - A\,\frac{\partial\tau/\partial y}{P} \tag{46}$$

Equation (46) reduces to the form used by Bernstein et al.[10] in the inviscid limit.

At the wall, $(1 - \overline{\tau_w'}) = 0$, which cancels the $1/M^2$ Singularity. Similarly, at some height above the wall, $(1 - \overline{\tau_w'}) \simeq 1$, and viscous effects become unimportant. At that point, the inviscid approximation is valid. In order to cancel the singularity, it is not sufficient merely to set $(1 - \overline{\tau_w'}) = 0$ at the wall. Rather, $(1 - \overline{\tau_w'}) \to 0$ as fast as $M^2 \to 0$ when $y \to 0$. For the calculations in Eq. (47) it will be assumed that $(1 - \overline{\tau_w'}) = 0$ is quadratic in height above the wall below the sonic line, above which it will be set to unity. This follows from the reasoning that pressure changes can be transmitted upstream only through the subsonic portion of the flow. In fact, numerical studies have shown that, because the sonic line is typically very near the wall in hypersonic boundary layers, the exact power of a polynomial used for this viscous function is unimportant, so long as it is sufficient to cancel the wall singularity.

In two dimensions, treating the streamtube area in Eq. (46) as an infinitesimal element and integrating across the channel height H, the streamwise derivative for the entire channel is

$$\frac{dH}{dx} = \frac{d\ell nP}{dx}\int_0^H\left[\frac{1}{M^2} - 1\right](1 - \overline{\tau_w'})\frac{dy}{\gamma} - \int_0^H \frac{\partial\tau/\partial y}{P}\,dy \tag{47}$$

where the uniform pressure assumption is used to pull the logarithmic derivative of pressure out of the integral.

There are two integral components. The first integral is identical to the inviscid relationship between area change and logarithmic pressure gradient, with a correction for viscosity. The second derivative represents the total shear on the wall and is entirely new in the viscous solution.

The compound compressible flow for continuous profile with shear is best characterized by a modified β parameter,

$$\beta = \int_0^H\left[\frac{1}{M^2 - 1}\right](1 - \overline{\tau_w'})\frac{dy}{\gamma} \tag{48}$$

With heat addition, the streamwise pressure gradient is related to the channel area as

$$\beta\,\frac{d\ell nP}{dx} = \left[\frac{dH}{dx} + \int_0^H \frac{\partial\tau/\partial y}{P}\,dy - \int_0^H\left(1 + \frac{\gamma - 1}{2}M^2\right)\frac{dT_o}{T_o}\,dy\right] \tag{49}$$

IV. Boundary-Layer Profile in the Compound Compressible Flow

Analytical Laminar Profile Solution

With a formulation for compound compressible flow that will allow a solution for a profile without wall slip, the behavior of a stratified compressible flow entering a channel can be examined, such as the forebody flow on a hypersonic vehicle coming into a scramjet inlet. To do so, an assumption must be made as to the nature of the Mach number profile and the viscous function in the wall region. The former will be input from the forebody boundary-layer profile; the latter will be inferred from the expected properties of a viscous shear layer.

It is most convenient to demonstrate the behavior with the laminar boundary layer because analytical similarity solutions are available. Following Cohen and Reshotko,[14] the laminar boundary-layer equations are solved in a transformed coordinate plane with transverse dimension η, which converts the problem to an incompressible one. Solving for the Mach number in terms of the tabulated stream function f, where $U = U_e f'(\eta)$ and the tabulated enthalpy function S,

$$M = \frac{U}{a} = \frac{U_e f'(\eta)}{\sqrt{\gamma RT}} \tag{50}$$

with the similarity relationship for temperature,

$$T = T_o\left\{[1 + S(\eta)] - \left(\frac{[(\gamma - 1)/2]\,M_e^2}{1 + [(\gamma - 1)/2]\,M_e^2}\right)f'^2(\eta)\right\} \tag{51}$$

$$M = \frac{U_e f'(\eta)}{\sqrt{\gamma RT_o\left\{[1 + S(\eta)] - \left(\dfrac{[(\gamma - 1)/2]\,M_e^2}{1 + [(\gamma - 1)/2]\,M_e^2}\right)f'^2(\eta)\right\}}} \tag{52}$$

Inverting the square of both sides of Eq. (52), and writing the freestream velocity in terms of Mach number and temperature,

$$\frac{1}{M^2} = \frac{(1 + [(\gamma - 1)/2]M^2)\left\{[1 + S(\eta)] - \left(\dfrac{[(\gamma - 1)/2]M_e^2}{1 + [(\gamma - 1)/2]M_e^2}\right)f'^2(\eta)\right\}}{M_e^2 f'(\eta)^2} \tag{53}$$

The value of $1/M^2$ in the similarity profile can then be used to solve for β through the boundary layer in Eq. (48). The compound flow parameter for the freestream flow will have to be added to this value to represent the entire inlet flowfield. This is the value of the compound flow parameter at one specific location, as the boundary layer begins to enter the channel. As the flow moves along the channel, the profile will change and new values for β must be derived.

For simplicity, the boundary-layer compound flow parameter can be evaluated under the assumption that the freestream Mach number is hypersonic ($M_e \to \infty$). Transforming to incompressible coordinates,

$$d\eta = \left[\frac{\displaystyle\int_0^{\eta_\delta}\frac{T}{T_o}\,d\eta}{\delta\left\{[1 + S(\eta)] - \left(\dfrac{\frac{\gamma - 1}{2}M_e^2}{1 + \frac{\gamma - 1}{2}M_e^2}\right)f'^2(\eta)\right\}}\right]dy \tag{54}$$

the boundary layer β parameter is

$$\int_0^H\left(\frac{1}{M^2} - 1\right)\{1 - \overline{\tau_w'}\}\frac{dy}{\gamma} = \frac{\delta}{\gamma}\int_0^{\eta_\delta}\left(\left[\frac{\left(\frac{\gamma - 1}{2}\right)\{[1 + S(\eta)] - f'^2(\eta)\}}{f'(\eta)^2} - 1\right] \times \frac{\delta\{[1 + S(\eta)] - f'^2(\eta)\}}{\eta_\delta + \int_0^{\eta_\delta} S(\eta)\,d\eta - \int_0^{\eta_\delta} f'^2(\eta)\,d\eta}\right)\{1 - \overline{\tau_w'}\}\,d\eta$$

The transformed integral expression for β is independent of Mach number at high Mach number. This is at first surprising, because the boundary-layer velocity is scaled with the freestream velocity, so that it might be suspected that an increase in freestream Mach number would increase Mach number throughout the boundary layer by a fixed proportion, thereby decreasing the value of $1/M^2$ by the square of that proportion. Because the boundary layer is compressible, however, density is sensitive to local temperature, which is in turn a function of the freestream conditions and the local Mach number. Increasing the freestream Mach number increases the temperature of the decelerated streamtubes with roughly the square of the Mach number, and thus decreases the density. This in turn effectively increases the area occupied by the low-speed streamtubes.

The boundary layer can be replaced by a uniform streamtube with the equivalent Mach number $\bar{M}$ defined earlier. In the hypersonic limit, $\bar{M} = 2.4$ for a cold wall and approaches $\bar{M} = 1$ for an adiabatic wall. This means that the laminar boundary layer will always behave like a uniform supersonic stream and that the entire channel flow must be compound supersonic. The equivalent Mach number for a laminar boundary layer is shown in Table 1 for several freestream Mach numbers.

Table 1 also shows the uniform equivalent Mach number for a turbulent boundary-layer profile as a function of freestream Mach number. A semi-empirical turbulent profile suggested by Perry and East[15] was used. In the turbulent case, the boundary-layer equivalent Mach number does indeed scale with the freestream Mach number and is generally close to the freestream value. The turbulent profile will obviously have a smaller effect on freestream properties than a laminar boundary layer of the same thickness.

V. Incorporating the Boundary Layer with the Freestream

Comparison to Two-Streamtube Model

The entire channel will be considered in terms of the relative thicknesses of the boundary layer and the inviscid freestream region. The boundary layer will be considered as a separate, "low-speed" streamtube, that is, at a lower speed than the freestream (the boundary layer may very well be mostly hypersonic), whereas the freestream is considered a uniform hypersonic layer.

In the compound compressible model, the effects of streamtubes on the net behavior of the entire channel flow can be added linearly because the β parameter is a linear function of streamtube area. The β parameter has dimensions of area, but it is most convenient to work with a nondimensionalized form of this parameter. If β is normalized to the entire channel height H, then, defining

$$\bar{\beta} = \frac{\beta}{A} \tag{55}$$

with a boundary-layer height of δ,

$$\bar{\beta}_{\text{channel}} = \left(\frac{\delta}{H}\right) \bar{\beta}_{\text{boundary layer}} + \left(1 - \frac{\delta}{H}\right) \bar{\beta}_{\text{freestream}} \tag{56}$$

The net channel flow can thus be parameterized in terms of the Mach number profile in the boundary layer, the freestream uniform-flow conditions, and the relative height of the boundary layer.

Table 1 Boundary-layer uniform equivalent Mach number $\overline{M}$ at various freestream Mach numbers, with freestream at 1000 K and wall cooled to 1800 K if that is less than adiabatic

	$\overline{M}$	
$M_{\text{freestream}}$	Laminar	Turbulent
5	1.41	2.89
10	1.76	4.62
15	1.94	5.39
20	2.05	6.27

With a uniform freestream,

$$\bar{\beta}_{\text{freestream}} = \frac{1}{\gamma}\left(\frac{1}{M_e^2} - 1\right) \tag{57}$$

and the normalized boundary-layer parameter is

$$\bar{\beta}_{\text{boundary layer}} = \int_0^1 \left[\frac{1}{M^2} - 1\right](1 - \overline{T_w'})\,\frac{\mathrm{d}y'}{\gamma} \tag{58}$$

where y' is the y coordinate normalized to boundary-layer height. The net Mach number $\overline{M}$, defined in Eq. (6), provides a useful means of comparing the channel with a boundary-layer profile to a one-dimensional channel with uniform flow. This is done in Fig. 4 for a laminar boundary layer and in Fig. 5 for a turbulent boundary layer. In both of these figures, the equivalent Mach number, which indicates how pressure is changing in the channel flow as a function of channel area change, is plotted as a function of the relative height of the ingested boundary layer, referenced to the channel height, for several values of freestream Mach number.

In Fig. 4, it is apparent that the laminar boundary layer is driving the channel flow to behave as if it were at a much lower Mach number than the actual freestream value, for even moderately thick ingested boundary-layer fractions. The turbulent profile has much less effect, as even channels with thick boundary layers behave as if they were near the freestream Mach number.

Numerical Iteration Scheme

It is important to emphasize that the results in the preceding subsection show the flow behavior at the inlet station only, just as the forebody boundary layer enters the engine, although this behavior indicates trends that will continue down

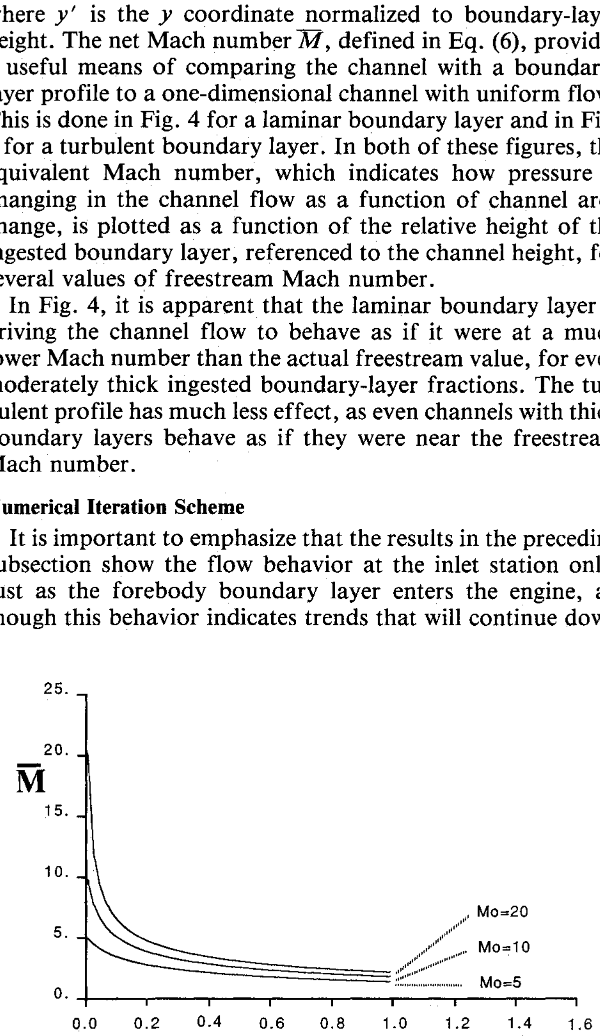

Fig. 4 Uniform equivalent Mach number $\bar{M}$ for the entire channel as a function of laminar boundary-layer thickness divided by channel height, δ/H.

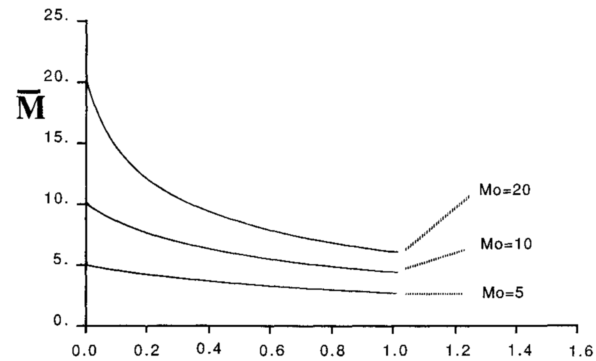

Fig. 5 Uniform equivalent Mach number $\bar{M}$ for the entire channel as a function of turbulent boundary-layer thickness divided by channel height, δ/H.

the channel by predicting the streamwise derivatives of Mach number, pressure, and temperature. From this equivalent Mach number, the streamwise derivatives of thermodynamic conditions and freestream Mach number can be determined at that location. Because $\overline{M}$ does not vary as if it were a uniform flow, the value of β must be recalculated at each station along the channel if the compound compressible flow model is to be applied throughout. This can be done numerically by updating the Mach number profile from the streamwise pressure gradient, which is determined directly from the local value of β and the channel area derivative, as per Eq. (3).

The streamwise change in Mach number at any point in the profile due to channel area is uncoupled from the rest of the profile. With heat addition,

$$\frac{dM}{dx} = -\left(1 + \frac{\gamma - 1}{2} M^2\right)\left[\frac{(1 - \overline{\tau'})}{\gamma M}\frac{d\ell nP}{dx} + \frac{M}{2}\frac{d\ell nT_o}{dx}\right] \tag{59}$$

The y coordinate of the Mach number profile is the position of each streamtube and is not fixed in absolute coordinates. The streamtube coordinates are superposed over a fixed coordinate system, designated here by Y, in which $dY/dx = 0$, but $dy/dx \neq 0$.

The increase in height of each streamtube is equal to the change in area of the streamtube extending from the wall to the particular height being examined. From Eq. (46),

$$\frac{dy}{dx} = \frac{1}{\beta_{\text{channel}}}\frac{d\ell nH}{dx}\int_0^Y\left(\left[\frac{1}{M^2} - 1\right](1 - \overline{\tau_w'})\frac{1}{\gamma} - \overline{\tau_w'}\right)dy + \left(1 + \frac{\gamma - 1}{2}M^2\right)\frac{dT_o}{T_o} \tag{60}$$

From this, a new profile of $M(y)$ can be determined, subject to the channel area changes and heating. This can then be used in the analytical formulation to determine the streamwise derivatives of the pertinent properties at the next station. This has been done in Ref. 5, where it was shown that the value of $\overline{M}$ tends to remain fairly constant in the channel, which corresponds to the insensitivity of $\overline{M}$ to freestream Mach number.

VI. Conclusions

The compound compressible flow model applied to hypersonic channel flows provides an analytical method for examining the influence of a thick-forebody boundary layer on the flow inside a hypersonic air-breathing engine. The compound flow model can be simplified greatly with the hypersonic approximations and demonstrates that a low-speed streamtube can have a strong impact on, or even dominate, the behavior of an adjacent hypersonic streamtube.

The simplest compound compressible flow model is a two-streamtube channel, in which one streamtube is hypersonic and the other is of lower speed. Such a flow is the most generic representation of the flow entering a hypersonic air-breathing engine with a thick boundary layer, such as may be encountered on a transatmospheric vehicle. The two-streamtube model can be worked into an analytical solution for simple combustor geometries, where it is found that the presence of a low-speed streamtube can produce demonstrable changes in thermodynamic conditions and freestream Mach number when compared to expected uniform flow properties. With an adequately thick subsonic stream, it was shown that a channel designed for uniform hypersonic flow can be compound-choked, or even compound subsonic.

The behavior of the two-stream model depends on whether the low-speed stream is subsonic, sonic, or supersonic, and how thick it is. An actual profile, such as that associated with an ingested boundary layer, will have a range of Mach numbers from zero at the wall, to the freestream value. Such a profile can be incorporated into a compound compressible flow model by dividing it into many small streamtubes and collapsing those streamtubes into integration elements. When that is done, the profile can be compared directly to a uniform streamtube. It has been found that both laminar and turbulent hypersonic boundary-layer profiles behave as if they were supersonic, so that the channel flow associated with an ingested hypersonic boundary layer must always be compound supersonic. This means that there is no danger of choking the flow at hypersonic flight conditions, as might be predicted from the simple two-stream analysis. However, it is also found that the laminar boundary layer is best represented by a nearly sonic Mach number, which can make the net behavior of a hypersonic channel flow resemble that of a lower Mach number. The turbulent boundary layer has a much smaller effect than the laminar one.

An important impact of the boundary layer inside a hypersonic engine may be the drop in static pressure and temperature of the freestream. The compound compressible flow model predicts that the presence of an ingested boundary layer will drop the pressure of the freestream below that associated with a uniform inflow. This in turn will reduce the combustion reaction rate, which will further drop the static pressure. This increase in reaction time may result in less heat being added in the combustor than is available from the chemical reaction or may require the design of a longer, and therefore heavier, combustor channel. It is therefore important that boundary-layer effects on the freestream be considered in combustor design and engine efficiency analysis.

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